CHAPTER 13

KIRCHHOFF'S LAWS

Electrical circuit problems solved so far in this course have contained only one source of e.m.f. In practice, however, a circuit may contain two or more sources, these being located in either a series or parallel branch of that circuit. One of the methods of solving problems of this nature is by the use of Kirchhoff's Laws which state —

(a) In any complete electrical circuit, the sum of the voltage rises must equal the sum of the voltage drops. This is known as the voltage law.

(b) At any junction point in an electrical circuit, the sum of the currents entering the junction must equal the sum of the currents leaving the junction.

13.1 KIRCHHOFF'S VOLTAGE LAW

The voltage law is used in the series part of a circuit. From an observation of a circuit, a current direction is assumed. All sources of e.m.f.'s which tend to cause current to flow in this direction are called voltage rises and given a positive sign. All sources of e.m.f.'s which tend to oppose current flow in the assumed direction, together with voltages appearing across resistors due to current flow, are said to be voltage drops. These are given a negative sign. A typical voltage series circuit is shown in Figure 13.1.

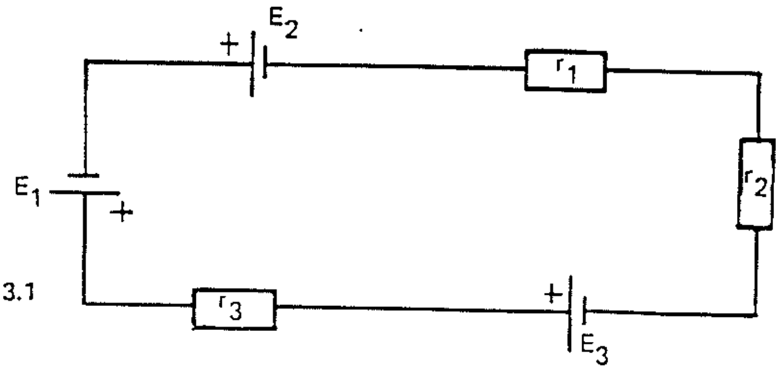


Figure 13.1

Assuming anitclockwise current, E_1 and E_2 are voltage rises, while E_3, Ir_1, Ir_2 and Ir_3 are voltage drops.

Example 13.1

Calculate the current in Figure 13.2.

Correct Diagram.

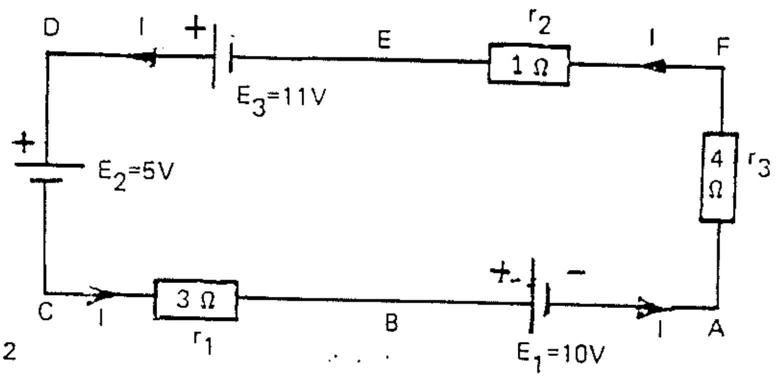


Figure 13.2

Around the loop A B C D E F (assuming Anti. clockwise current)

$$E_1 - Ir_1 + E_2 - E_3 - Ir_2 - Ir_3 = 0$$

$$(E_1 + E_2 - E_3) - (Ir_1 - Ir_2 + Ir_3) = 0$$

$$E_1 - E_2 - E_3 = Ir_1 + Ir_2 + Ir_3$$

$$E_1 - E_2 - E_3 = I(r_1 - r_2 - r_3)$$

$$10 + 5 - 11 = I(3 + 1 + 4)$$

$$4 = 8I$$

$$I = 0.5\ A$$

Example 13.2

If the voltage drop across R_1 in figure 13.3 is 12 volts determine the value of E_2.

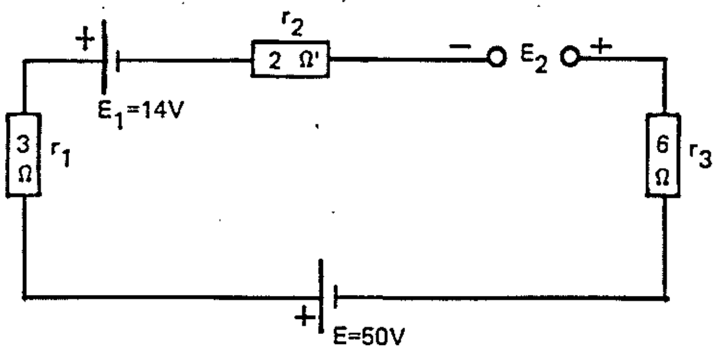


Figure 13.3

Assume clockwise direction of current

$$I = \frac{12}{3} = 4A$$

$$E - Ir_1 - E_1 - Ir_2 + E_2 - Ir_3 = 0$$

$$50 - 12 - 14 - 8 + E_2 - 24 = 0$$

$$E_2 - 58 + 50 = 0$$

$$E_2 - 8 = 0$$

$$E_2 = 8\ V$$

Example 13.3

When the switch in figure 13.4 is closed a current of 0.5 amperes passes through the circuit. Use Kirchhoff's voltage law to calculate the value of the unknown resistor R_4.

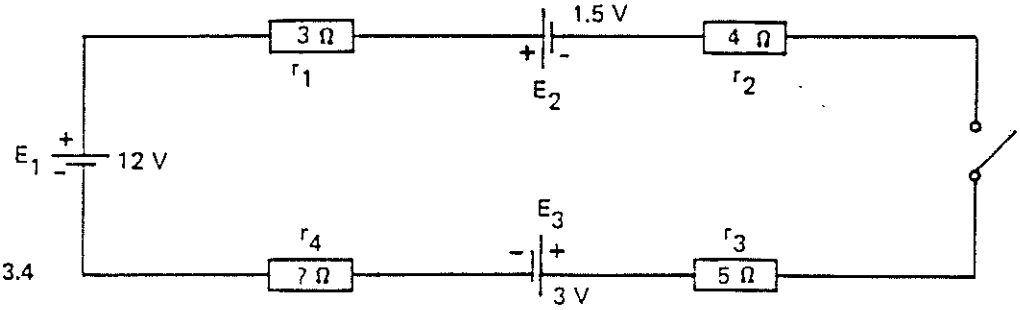


Figure 13.4

Using Kirchhoff's voltage law around the circuit. (Assume clockwise current direction).

$$+ (E_1) + (-Ir_1) + (-E_2) + (-Ir_2) + (-Ir_3) + (-E_3) + (-Ir_4) = 0$$

removing the brackets

$$E_1 - Ir_1 - E_2 - Ir_2 - Ir_3 - E_3 - Ir_4 = 0$$

Substituting values

$$12 - (0.5 \times 3) - 1.5 - (0.5 \times 4) - (0.5 \times 5) - 3 - (0.5 \times r_4) = 0$$

$$12 - 1.5 - 1.5 - 2.0 - 2.5 - 3 - 0.5r_4 = 0$$

$$12 - 10.5 - 0.5r_4 = 0$$

$$0.5r_4 = 1.5$$

$$r_4 = 3\ \Omega$$

13.2 KIRCHHOFF'S CURRENT LAW

In the current law, all current entering a junction are given a positive sign, while all current leaving the junction are given a negative sign. (Figure 13.5)

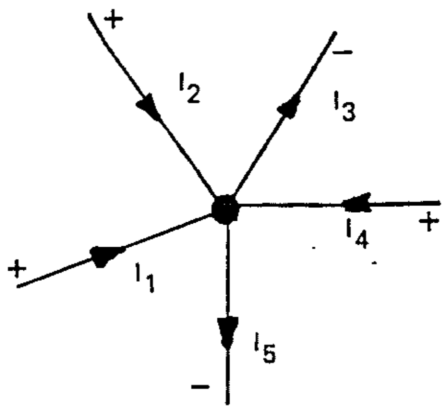


Figure 13.3

The law states that the algebraic sum of the currents at the junction is zero. Taking into account the sign of each current, the current equation for Figure 13 becomes –

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0$$

Example 13.4

Determine the current I_6 and its direction in figure 13.6.

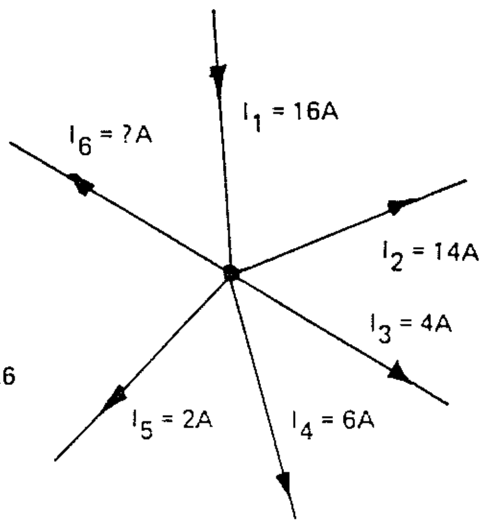


Figure 13.6

Assume the direction of I_6 is outwards. The current equation for the junction becomes:-

$$+ (I_1) + (-I_2) + (-I_3) + (-I_4) + (-I_5) + (-I_6) = 0$$

Removing the brackets

$$I_1 - I_2 - I_3 - I_4 - I_5 - I_6 = 0$$

Substituting values

$$16 - 14 - 4 - 6 - 2 - I_6 = 0$$

$$- I_6 = 10$$

$$I_6 = -10A$$

If there is a difference in the algebraic sign in the answer the wrong direction of current has been assumed. Reverse the assumed direction and check answers.

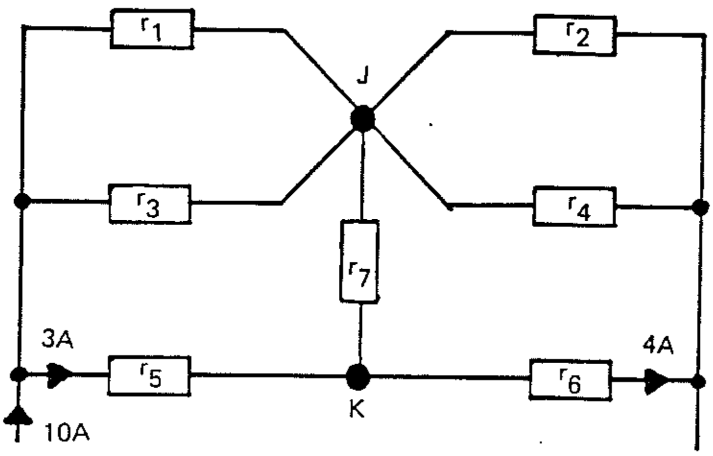
$$I_1 + I_2 - I_3 - I_4 - I_5 + I_6 = 0$$

$$16 + 14 - 4 - 6 - 2 + 10 = 0 \quad \text{(correct)}$$

Example 13.5

In figure 13.6 it is known that r_1 and r_3 are equal in value and r_2 is twice the value of r_4. Determine the magnitude and direction of the currents at junction J.

Figure 13.7



In figure 13.7

$$I_1 + I_2 = 10 - 3$$

$$= 7\text{A}$$

$$r_1 = r_3$$

so – $I_1 = I_3 = 3.5$ A (into junction)

at junction K –

$$I_5 + I_7 + I_6 = 0$$

$$3 + I_7 - 4 = 0$$

$$I_7 = 1 \text{ A (into junction)}$$

at junction J –

$$I_1 + I_3 + I_7 + I_2 + I_4 = 0$$

$$3.5\text{(IN)} + 3.5\text{(IN)} - 1\text{(OUT)} + I_2 + I_4 = 0$$

$$I_2 + I_4 = -6 \text{ A}$$

$$2r_2 = r_4$$

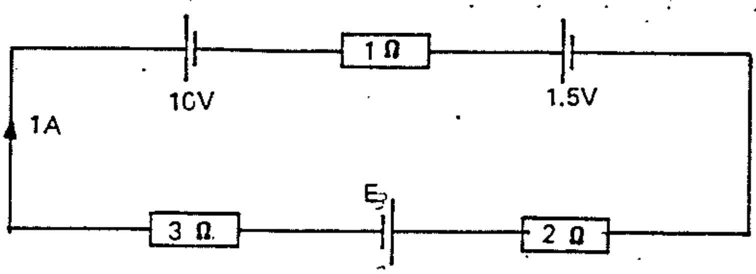
so current divides in the ratio of 2.1

$$I_2 = 2 \text{ A (OUT)}$$

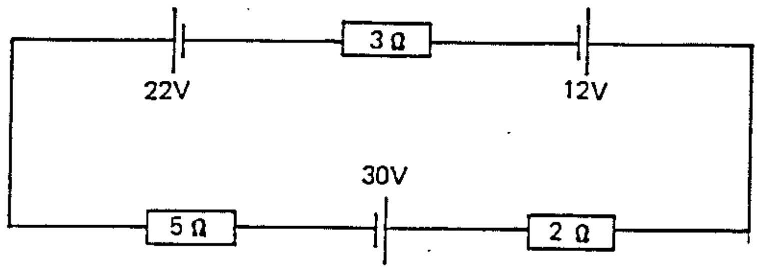
$$I_4 = 4 \text{ A (OUT)}$$

TUTORIAL 13

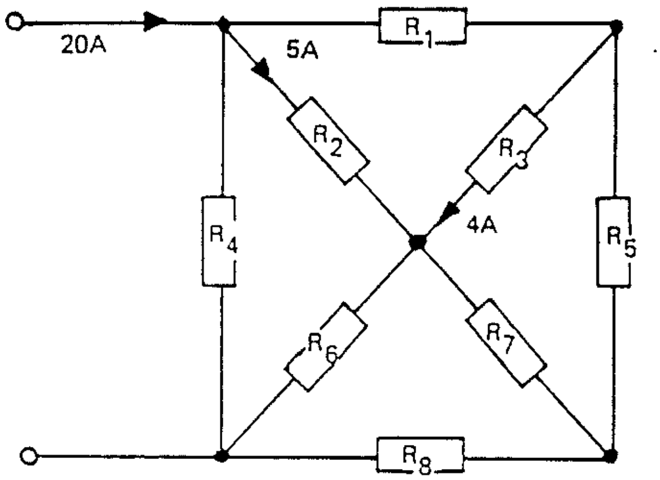
(1) Calculate the terminal potential difference of E_3 in the following circuit.



(2) Calculate the current in the following circuit.



(3) In the following circuit R_1 is equal to R_4 while R_6 is equal to R_7. Determine the currents and their directions in all the resistors in the circuit.



(4) In the following circuit R_1 and R_3 are equal values. The total resistance of the circuit is 10 Ω Determine all the currents and the directions in each of the resistors in the circuit.